BM – 101 Algebra and Trigonometry

Assignment – I

Note: Attempt any five.

1. Prove that necessary and sufficient condition for a matrix A to be Hermitian is that $A^{\theta} = A$.

2. Show that every square matrix can be written as one and only one way as a sum of symmetric and skew-symmetric matrices.

3. If $A_{n \times n}$ be a square matrix of rank $n-1$, then prove that $\text{adj}\, A \neq 0$.

4. Prove that the rank of the product of two matrices cannot exceed the rank of either matrix.

5. Prove that if two vectors are L.D., then one of them is the scalar multiple of other:

6. Find the eigne ~~value~~ vectors of the matrix $A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$

7. State and prove Cayley – Hamilton theorem.

8. Prove that characterstics roots of a skew-symmetric matrix are either ~~purely~~ zero or purely imaginary.

BM-101 Alegebra and Trigonometry

Assignment — II

Note : Attempt any five.

1. Prove that the set $G = \{a + b\sqrt{2} : a, b \in Q\}$ is an infinite abelian group w.r.t. addition.

2. Prove that in a group, identity element is unique. Also prove that every element has unique inverse.

3. If H and K are two subgroup of a group G, then HK is a subgroup of G iff $HK = KH$.

4. Prove that number of generators of a finite cyclic group of order n is $\phi(n)$, wher $\phi(n)$ is Euler's ϕ function.

5. A subgroup H of a group G is a normal iff $xhx^{-1} \in G \ \forall \ h \in H, x \in G$.

6. State & prove fundamental theorem of Homomorphism.

7. Prove that every quotient group of a cyclic group is cyclic

8. Prove that a non-abelian group has atleast one non-trivial inner automorphism.

BM - 102 Calculus

Assignment - I

Note: Attempt any five.

1. If $f(x) = \begin{cases} 0 & \text{if } x \in R - Q \\ 1 & \text{if } x \in Q \end{cases}$ Then find $\lim_{x \to a} f(x)$, where $a \in R$.

2. Prove that (i) $\lim_{\theta \to 0} \frac{\sin \theta^\circ}{\theta} = \frac{\pi}{180^\circ}$

(ii) $\lim_{x \to 0} \frac{\sin x - \cos x}{x - \pi/4} = \sqrt{2}$.

3. State and prove Sandwich Theorem for limit.

4. By using $\epsilon-\delta$ definition that $f(x) = \begin{cases} \sin\frac{1}{x} & x \neq 0 \\ 0 & x = 0 \end{cases}$ is discontinuous at $x = 0$.

5. Prove that $f(x) = x + |x|$ is continuous at $x = 0$ but not derivable at $x = 0$.

6. If $y = x^n e^{ax}$ & $y_2 - 4y_1 + 4y = 0$, find n & a.

7. Evaluate $\int_0^{\pi/2} \sin^n x \, dx$, where n is positive integer greater then one.

8. Solve $\int_0^1 \frac{x^3}{\sqrt{1-x^2}} dx$.

BM- 102 Calculus

Assignment - II

Note : attempt any five.

1. Find the necessary and sufficient condition that the equation $M\,dx + N\,dy = 0$ may be exact.

2. Solve $(x^2+y^2)\,dx - 2xy\,dy = 0$

3. Solve $(x^2+y^2+1)\,dx - 2xy\,dy = 0$

4. Solve $p^3 + 2xp^2 - y^2p^2 - 2xy^2p = 0$, $p = \frac{dy}{dx}$

5. Solve $y = 2px + p^4x^2$, $p = \frac{dy}{dx}$

6. Solve and find the complete primitive and singular solution of the equation $3y = 2px - \frac{2p^2}{x}$ where $p = \frac{dy}{dx}$.

7. Find the orthogonal trajectories of the family of parabolas $y = ax^2$.

8. Find the orthogonal trajectories of the cardioid $r = a(1-\cos\theta)$, a is parameter.

BM - 103 Vector Analysis and Geometry

Assignment - I

Note: attempt any five.

1. The necessary and sufficient condition that three non-zero, non-parallel vectors $\vec{a}, \vec{b}, \vec{c}$ is $[\vec{a}\ \vec{b}\ \vec{c}] = 0$

2. Evaluate (i) $\hat{i} \cdot (\hat{j} \times \hat{k})$
   (ii) $\hat{i} \cdot (\hat{j} \times \hat{k}) + (\hat{i} \times \hat{k}) \cdot \hat{j}$

3. Let $\vec{r}$ be the vector function of a scalar t and $|\vec{r}| = r$ and $\vec{a}, \vec{b}$ are constant vectors when differentiate the following w.r.t 't' (i) $r^n \vec{r}$ (ii) $\vec{r}^2 + \frac{1}{\vec{r}^2}$

4. The necessary and sufficient ~~function~~ condition for a vector function $\vec{f}$ of a scalar variable t to be constant is $\frac{d\vec{f}}{dt} = 0$

5. Find $\frac{d^2\vec{r}}{dt^2}$ ~~when~~ Given that $\vec{r} = 3\hat{i} - 6t^2\hat{j} + 4t\hat{k}$

6. Prove that $\nabla(\phi + \psi) = \nabla\phi + \nabla\psi$

7. Prove that $\nabla \cdot (\phi\vec{f}) = (\nabla\phi) \cdot \vec{f} + \phi(\nabla \cdot \vec{f})$

8. If $\phi = x^2 y^3 z^4$. Find div(grad ϕ).

BM - 103

Vector Analysis and Geometry.

Assignment - II

Note: Attempt any five.

1. Trace the curve $x^2(y^2+a^2)+y^2(y^2-a^2)=0$ where a is constant.

2. Trace the curve $y^3 = x^3 + ax^2$.

3. Find the equation of a conic which double contact with a given conic.

4. Prove that the conics $x^2+2y^2-1=0$ and $3x^2+8xy+10y^2-4x-8y+1=0$ have double contact with each other.

5. Find the conics confocal with $x^2+2y^2=2$ which pass through $(1,1)$.

6. Find the locus ~~&~~ of mid-point of the chords of the confocal $\frac{x^2}{a^2+\lambda}+\frac{y^2}{b^2+\lambda}=1$ which touch the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$.

7. Find the equation of the tangent planes to $7x^2-3y^2-z^2+21=0$ which pass through the line $7x-6y+9=0$, $z=3$

8. Find the equation of director sphere of the